

INSTITUTO TECNOLÓGICO DE AERONÁUTICA  
MP-208: Optimal Filtering with Aerospace Applications  
Computational Exercise 1

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Consider an MAV equipped with an ultrasonic sensor that provides a set  $y_{1:N}$  of  $N$  measures of some physical quantity. Assume that  $y_i \in \mathbb{R}$  is modeled by

$$y_i = \mathbf{h}_i \boldsymbol{\theta} + v_i, \quad i = 1, \dots, N,$$

where  $\mathbf{h}_i \triangleq [i \ 1]$ ,  $\boldsymbol{\theta} \triangleq [\theta_1 \ \theta_2]^T \in \mathbb{R}^2$  is a realization of  $\boldsymbol{\Theta} \sim \mathcal{N}(\mathbf{m}_{\boldsymbol{\Theta}}, \mathbf{P}_{\boldsymbol{\Theta}})$ ,  $v_i \in \mathbb{R}$  is a realization of  $V_i \sim \mathcal{N}(0, R)$ ,  $\{V_i\}$  is a white sequence, and  $V_i$  and  $\boldsymbol{\Theta}$  are uncorrelated. Consider that  $\mathbf{m}_{\boldsymbol{\Theta}} = [1 \ 2]^T$ ,

$$\mathbf{P}_{\boldsymbol{\Theta}} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.04 \end{bmatrix},$$

and  $R = 0.01$ .

- a.* Implement in a MATLAB script an LS estimator for  $\boldsymbol{\theta}$  from  $y_{1:N}$ . Adopt a constant weight  $W_i = 1, \forall i$ . Conduct a Monte Carlo simulation with 100 realizations of  $\hat{\boldsymbol{\Theta}}_N$ . Obtain the sample mean and RMSE of the realizations. Present a histogram of the estimate realizations.
- b.* Implement in a MATLAB script an MAP estimator for  $\boldsymbol{\theta}$  from  $y_{1:N}$ . Conduct a Monte Carlo simulation with 100 realizations of  $\hat{\boldsymbol{\Theta}}_N$ . Obtain the sample mean and RMSE of the realizations. Compute the theoretical mean and RMSE (square root of MSE). Present an histogram of the estimate realizations.
- c.* Compare the sample means and RMSEs obtained in *a* and *b*.
- d.* Present a succinct report answering all the above questions.