INSTITUTO TECNOLÓGICO DE AERONÁUTICA
MP-208: Optimal Filtering with Aerospace Applications
Computacional Exercise 2

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Consider a discrete-time second-order dynamic system described, with a sampling time of $T = 0.1$ s, by:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k + \mathbf{w}_k$$

$$y_{k+1} = \mathbf{C}\mathbf{x}_{k+1} + v_k$$

where $\mathbf{x}_k \in \mathbb{R}^2$ is the state vector, $y_k \in \mathbb{R}$ is the measured output, $\left\{\mathbf{w}_k \in \mathbb{R}^2\right\}$ is a realization of the SP $\{\mathbf{W}_k\}$, with $\mathbf{W}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ and $\mathbf{Q} = \text{diag}(1 \times 10^{-2}, 4 \times 10^{-2})$, $\{v_k \in \mathbb{R}\}$ is a realization of the SP $\{V_k\}$, with $V_k \sim \mathcal{N}(\mathbf{0}, R)$ and $R = 1 \times 10^{-2}$, $\mathbf{x}_1 \in \mathbb{R}^2$ is a realization of the RV $\mathbf{X}_1 \sim \mathcal{N}(\bar{\mathbf{x}}, \bar{\mathbf{P}})$, with $\bar{\mathbf{x}} = [1\ 0]^{\mathrm{T}}$ and $\bar{\mathbf{P}} = \text{diag}(1 \times 10^{-4}, 1 \times 10^{-8})$, the sequence $\left\{\{V_k\}, \{\mathbf{w}_k\}, \mathbf{X}_1\right\}$ is uncorrelated, $u_k \in \mathbb{R}$ is a control input[1]

$$u_k = 10\left(\bar{y}_k - \mathbf{e}_1^{\mathrm{T}}\mathbf{x}_k\right) - 2\mathbf{e}_2^{\mathrm{T}}\mathbf{x}_k,$$

in which $\bar{y}_k \in \mathbb{R}$ is the command input, consider $\bar{y}_k = 5$, $\forall k$, and finally,

$$\mathbf{A} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

a. Simulate the above system in the period from 0 to 20 s. Obtain a unique plot containing 10 realizations $\{y_k\}$ of the measured output together with the input command $\{\bar{y}_k\}$.

b. Using the true parameters (of the given model), implement a conventional Kalman filter for this system. Simulate the filter on all the ten measurement realizations $\{y_k\}$ obtained in *a*. Obtain two separate plots, one for $i = 1$ and the other for $i = 2$, with the ten realizations of the (true) estimation error $\mathbf{e}_i^{\mathrm{T}}\tilde{\mathbf{x}}_{k|k}$ and the respective sample means and RMS values, as well as the theoretical standard deviations (calculated from $\mathbf{P}_{k|k}$).

[1] $\mathbf{e}_1 \triangleq [1\ 0]^{\mathrm{T}}$, $\mathbf{e}_2 \triangleq [0\ 1]^{\mathrm{T}}$.