INSTITUTO TECNOLÓGICO DE AERONÁUTICA
MP-208: Optimal Filtering with Aerospace Applications
Computational Exercise 3

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May 14, 2019

Consider a system described by:

$$\dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), u(t)\big) + \mathbf{w}(t),$$

$$y_{k+1} = h\left(\mathbf{x}_{k+1}\right) + v_{k+1},$$

where $\mathbf{x}(t) \triangleq [x_1(t)\ x_2(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is the state vector at the continuous time t, $\mathbf{x}_k \triangleq \mathbf{x}(t_k)$ is the state vector at the discrete time k, $u(t) \in \mathbb{R}$ is the control input at the continuous time t, $\{\mathbf{w}(t) \in \mathbb{R}^2\}$ is the state noise, $y_k \in \mathbb{R}$ is the measured output at the discrete time k, $\{v_k \in \mathbb{R}\}$ is the measurement noise, and

$$\mathbf{f}(\mathbf{x}(t), u(t)) \triangleq \begin{bmatrix} -x_1(t) + x_2(t) \\ -0.1x_1(t)^2 - 1 + u(t) \end{bmatrix}, \tag{1}$$

$$h(\mathbf{x}) = x_1. \tag{2}$$

For the sake of convenience, consider the control input

$$u(t) = -10y(t) + 10, \tag{3}$$

where $y(t) \in \mathbb{R}$ is the continuous-time version of y_k.

Adopt the parameters presented in Table 1.

Question 1. Simulate the system described above using a Simulink diagram.

Question 2. Design and implement (in a MATLAB script) a CDEKF to estimate $\{\mathbf{x}(t)\}$. This script has to contain a Monte Carlo loop with an arbitrary number of realizations. Moreover, it has to read and process the simulated measures and true states from the Simulink diagram (Question 1) sequentially.

Question 3. Implement another MATLAB script like in Question 2, but using the CDUKF instead of the CDEKF.

Question 4. Using the MATLAB scripts and Simulink diagram of the above questions, conduct a simulation study comparing the performance of the CDEKF and CDUKF over 100 realizations.

Table 1: System parameters.

Description	**Value**
Covariance of the state noise	$\mathbf{Q}(t) = 0.01\mathbf{I}_2$
Covariance of the measurement noise	$R_k = 0.01$
Initial state	$\bar{\mathbf{x}} = \mathbf{0}_2$, $\bar{\mathbf{P}} = \mathbf{I}_2$
Sensor sampling time	$T_s = 0.1$ s