MP-282
Dynamic Modeling and Control of Multirotor Aerial Vehicles
Chapter 1: Introduction

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Contents

Dynamic modeling . . .

Notation

Geometric and Algebraic Vectors:

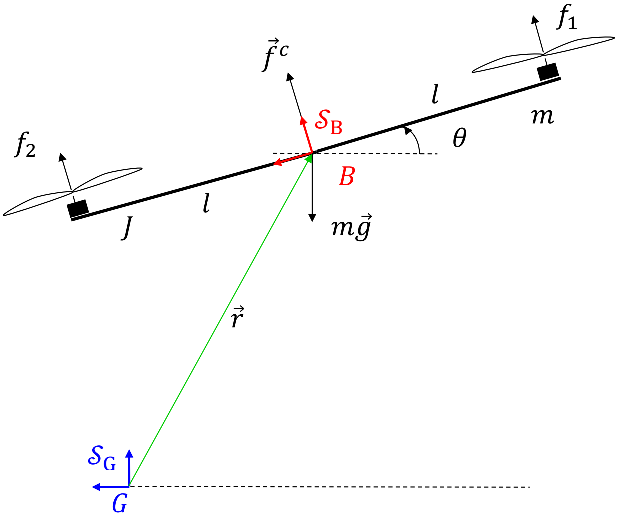
- $\overrightarrow{a}$: geometric vector
- $\hat{a}$: unit geometric vector
- $\mathcal{S}_{\mathrm{B}} \triangleq \{B; \hat{x}_{\mathrm{B}}, \hat{y}_{\mathrm{B}}, \hat{z}_{\mathrm{B}}\}$: body Cartesian coordinate system (CCS)
- $\mathcal{S}_{\mathrm{G}} \triangleq \{G; \hat{x}_{\mathrm{G}}, \hat{y}_{\mathrm{G}}, \hat{z}_{\mathrm{G}}\}$: ground CCS
- $\mathbf{a}_{\mathrm{B}}$: representation of $\overrightarrow{a}$ in $\mathcal{S}_{\mathrm{B}}$ (algebraic vector)
- $\bar{\mathbf{a}}_{\mathrm{B}}$: is a command for $\mathbf{a}_{\mathrm{B}}$
- $\mathbf{D}^{\mathrm{B/G}}$: attitude matrix of $\mathcal{S}_{\mathrm{B}}$ w.r.t. $\mathcal{S}_{\mathrm{G}}$
- $\mathbf{e}_1 \triangleq [1\ 0]^{\mathrm{T}}$ and $\mathbf{e}_2 \triangleq [0\ 1]^{\mathrm{T}}$

Some properties of $\mathbf{D}^{\mathrm{B/G}} \in \mathrm{SO}(3)$ (see more in Chapter 2):

$$\mathbf{a}_{\mathrm{B}} = \mathbf{D}^{\mathrm{B/G}} \mathbf{a}_{\mathrm{G}}$$

$$\left(\mathbf{D}^{\mathrm{B/G}}\right)^{\mathrm{T}} = \left(\mathbf{D}^{\mathrm{B/G}}\right)^{-1} = \mathbf{D}^{\mathrm{G/B}}$$

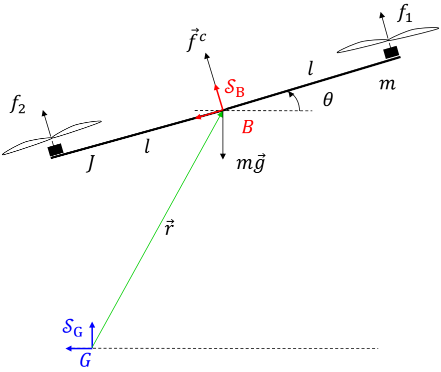
A Fictitious MAV in 2D



Translational Motion

Using the Newton's Second Law, the MAV's translational motion can be described in $\mathcal{S}_{\mathrm{G}}$ by

$$\ddot{\mathbf{r}}_{\mathrm{G}} = -g\mathbf{e}_2 + \frac{1}{m}\mathbf{D}^{\mathrm{G/B}}(f_1 + f_2)\mathbf{e}_2 \qquad (1)$$



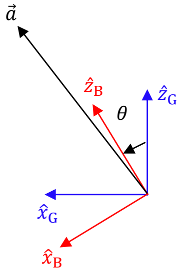
Rotational (Attitude) Motion

Using the Euler's Second Law, the MAV's attitude motion is described in $\mathcal{S}_{\mathrm{B}}$ (or wherever, since there is just one rotational DOF) by

$$\ddot{\theta} = \frac{1}{J}\tau^{c} \quad (2)$$

The relation between θ and the attitude matrix $\mathbf{D}^{\mathrm{B/G}}$ (whose transpose appears in equation (1)) is

$$\mathbf{D}^{\mathrm{B/G}} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (3)$$

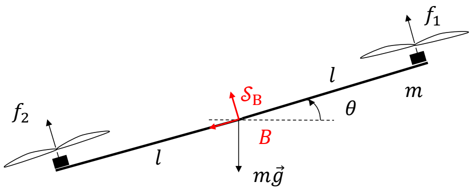


Rotor Configuration

By the geometry in the illustration, we see that the relationship between the individual thrust force magnitudes f_1 and f_2 and the resulting efforts (force and torque) is:

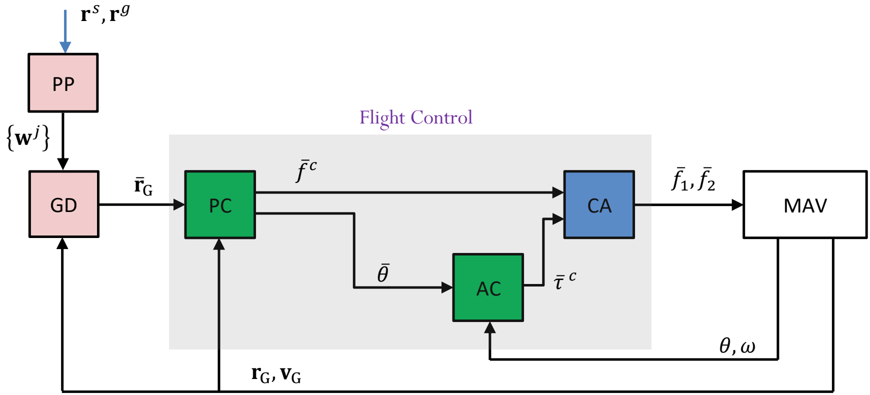
$$\begin{bmatrix} f^c \\ \tau^c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ l & -l \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (4)$$

where $f^c \triangleq f_1 + f_2$.



Flight Control . . .

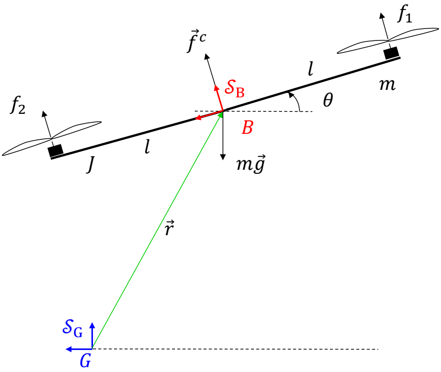
A Hierarchical Control Scheme



Abbreviations: PP (Path Planning)
GD (Guidance)
PC (Position Controller)
AC (Attitude Controller)
CA (Control Allocation)

Time-Scale Separation

- Assume that AC is much faster than PC
- In this case, AC and PC can be designed separately



Attitude Controller

Consider the design model

$$\ddot{\theta} = \frac{1}{J}\bar{\tau}^c \tag{5}$$

Let us adopt a simple saturated proportional-derivative attitude control law:

$$\bar{\tau}^c = \mathrm{sat}_{[-\tau^{\max},\tau^{\max}]}\left(JK_1\left(\bar{\theta} - \theta\right) - JK_2\dot{\theta}\right) \tag{6}$$

From (5)–(6), if the saturation is not active, one can approximately describe the closed-loop attitude dynamics by:

$$\ddot{\theta} + K_2\dot{\theta} + K_1\theta = K_1\bar{\theta} \tag{7}$$

Remark: From equation (7), one can easily design K_1 and K_2, *e.g.*, for given specifications of t_p (peak instant) and M_P (overshoot).

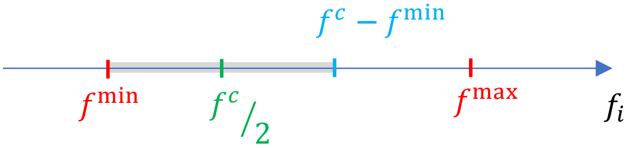
Attitude Controller

Assume that $f^{\min} \leq f_1, f_2 \leq f^{\max}$, $f^{\max} > f^c$, and $f^{\min} > 0$. Therefore, in order to respect the actuation symmetry of f_1 and f_2 with respect to $f^c/2$ (why?), one must assure that

$$f_1, f_2 \in \left[f^{\min}, f^c - f^{\min}\right] \tag{8}$$

Consequently, the maximal torque bound is

$$\tau^{\max} = \left(f^c - 2f^{\min}\right) l \tag{9}$$



Position Controller

Consider the design model

$$\ddot{\mathbf{r}}_{\mathrm{G}} = -g\mathbf{e}_2 + \frac{1}{m}\bar{\mathbf{f}}_{\mathrm{G}}^{c} \tag{10}$$

Let us adopt a simple saturated proportional-derivative position control law:

$$\bar{\mathbf{f}}_{\mathrm{G}}^{c} = \mathrm{sat}_{[\mathbf{f}^{\min},\mathbf{f}^{\max}]}\left(m\mathbf{K}_3\left(\bar{\mathbf{r}}_{\mathrm{G}} - \mathbf{r}_{\mathrm{G}}\right) - m\mathbf{K}_4\dot{\mathbf{r}}_{\mathrm{G}} + mg\mathbf{e}_2\right) \tag{11}$$

From (10)–(11), if the saturation is not active, one can approximately describe the closed-loop position dynamics by:

$$\ddot{\mathbf{r}}_{\mathrm{G}} + \mathbf{K}_4\dot{\mathbf{r}}_{\mathrm{G}} + \mathbf{K}_3\mathbf{r}_{\mathrm{G}} = \mathbf{K}_3\bar{\mathbf{r}}_{\mathrm{G}} \tag{12}$$

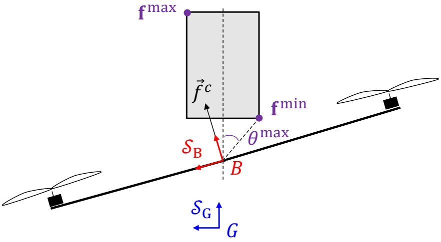
Remark: Just like the attitude controller, one can easily design $\mathbf{K}_3$ and $\mathbf{K}_4$ for given t_p and M_P. Compared to the attitude control design, here t_p must be much larger to respect the time-scale separation assumption!

Position Controller

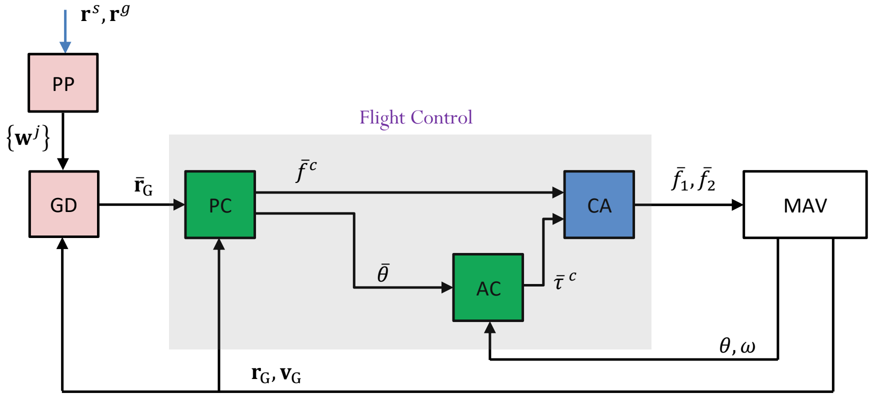
In order to prevent large inclinations, rotor turn-off, and actuator saturation, one can choose the force bounds:

$$\mathbf{f}^{\mathrm{min}} = \begin{bmatrix} -2f^{\mathrm{min}} \tan\theta^{\mathrm{max}} \\ 2f^{\mathrm{min}} \end{bmatrix} \quad (13)$$

$$\mathbf{f}^{\mathrm{max}} = \begin{bmatrix} 2f^{\mathrm{min}} \tan\theta^{\mathrm{max}} \\ 2f^{\mathrm{max}} \end{bmatrix} \quad (14)$$



A Hierarchical Control Scheme



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Control Allocation

In this simple example, by inverting equation (4) and replacing the efforts by the respective commands, we obtain the control allocation for the rotors:

$$\begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/(2l) \\ 1/2 & -1/(2l) \end{bmatrix} \begin{bmatrix} \bar{f}^c \\ \bar{\tau}^c \end{bmatrix} \tag{15}$$

where $\bar{f}^c \triangleq \|\bar{\mathbf{f}}^c_{\mathrm{G}}\|$.

On the other hand, the control allocation for the inner loop can be obtained from:

$$\bar{\theta} = \operatorname{atan} \frac{\mathbf{e}_1^{\mathrm{T}} \bar{\mathbf{f}}^c_{\mathrm{G}}}{\mathbf{e}_2^{\mathrm{T}} \bar{\mathbf{f}}^c_{\mathrm{G}}} \tag{16}$$

Flight Planning . . .

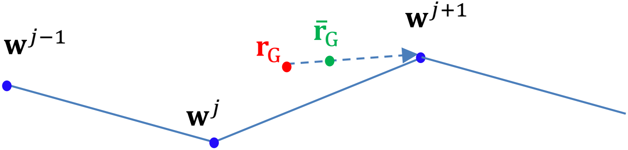
Guidance and Trajectory Planning

Some characteristics:

- Generate a desired trajectory $\bar{\mathbf{r}}_{\mathrm{G}}$ from a sequence of waypoints $\{\mathbf{w}^j\}$.
- It can be as simple as a proportinal control law that guides the MAV towards the next waypoint of the sequence:

$$\bar{\mathbf{r}}_{\mathrm{G}} = \mathbf{r}_{\mathrm{G}} + \mathbf{K}^g \left(\mathbf{w}^j - \mathbf{r}_{\mathrm{G}}\right) \tag{17}$$

where $\mathbf{K}^g$ is a gain (diagonal) matrix.



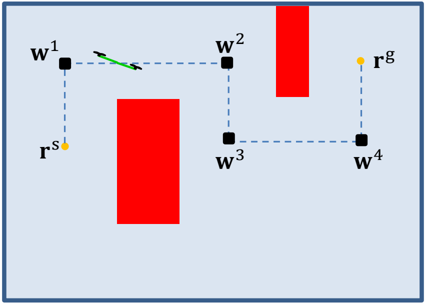
Guidance and Trajectory Planning

Here we have a simplified algorithm for the above guidance law:

Data: $\{\mathbf{w}^j\}$
$j \leftarrow 1$
for $k = 1 : end$ **do**
    $\mathbf{r}_{\mathrm{G}}(k) \leftarrow readInput$
    if $\mathbf{r}_{\mathrm{G}}(k) \in \mathcal{B}(\mathbf{w}^j)$ **then**
        $j \leftarrow j + 1$
    end
    $\bar{\mathbf{r}}_{\mathrm{G}}(k+1) \leftarrow \mathbf{r}_{\mathrm{G}}(k) + \mathbf{K}^g * (\mathbf{w}^j - \mathbf{r}_{\mathrm{G}}(k))$
    $\dots$
end

Path Planning

- It aims at computing an obstacle-free path, generally consisting of a sequence of waypoints $\mathbf{w}^j$, from the starting point $\mathbf{r}^s$ to the goal point $\mathbf{r}^g$.
- There are many methods to make it automatically (*e.g.*, visibility graph, PRM, RRT, BIT, etc.).
- In a very simple scenario, one could manually choose the apparently shortest path which is as far as possible to the obstacles, *e.g.*,



Thanks!