MP-282
Dynamic Modeling and Control of Multirotor Aerial Vehicles
Chapter 2: External Efforts and Actuators

Prof. Dr. Davi Antônio dos Santos
Instituto Tecnológico de Aeronáutica
www.professordavisantos.com

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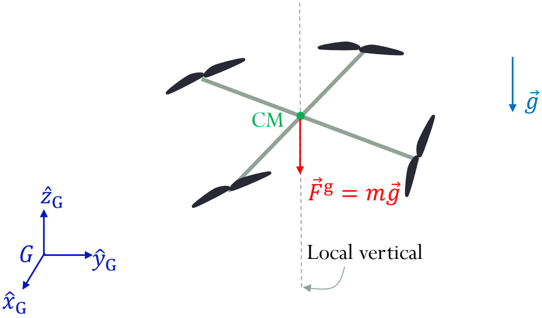
Contents

External efforts . . .

Gravity

Physical Model

- Considering MAVs with symmetrical structures, it is reasonable to neglect any torque around the center of rotation due to gravity.
- The figure below illustrates an MAV with mass m and the gravity force $\overrightarrow{F}^g$ acting on its center of mass (CM).



Gravity

Algebraic Representation in $\mathcal{S}_{\mathrm{G}}$

By considering that, in the flight space of interest, the gravity acceleration is uniform, its magnitude is $g \in \mathbb{R}$, and it is aligned with the negative direction of the local vertical, the $\mathcal{S}_{\mathrm{G}}$ representation of $\overrightarrow{F}^{g}$ is

$$\mathbf{F}^{g}_{\mathrm{G}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \quad (1)$$

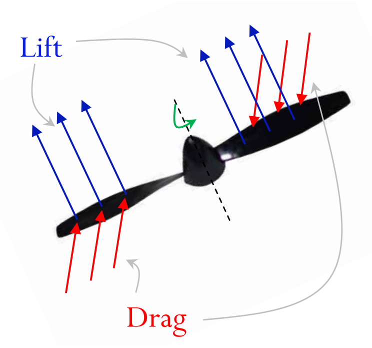
One can say that this model describes the Earth as non-rotating and plain! But this is sufficient for many MAV missions, with relatively short duration.

Aerodynamics

Propeller

In general, the rotation of a propeller in the air generates two aerdynamics forces:

- Lift: parallel to the rotor axis.
- Drag: orthogonal to the rotor axis.



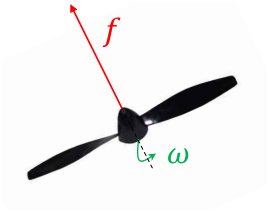
Aerodynamics

Rotor Thrust

The resulting lift is called here the rotor thrust and is modeled by

$$f = k_f \omega^2 \tag{2}$$

where $\omega \in \mathbb{R}$ is the rotor angular speed and k_f is the force coefficient.



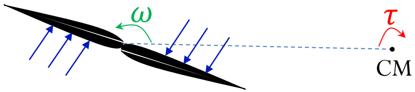
Remark: The aerodynamic coefficient k_f depends on the air density, the propeller geometry, and the flow regime. Fortunately, it can be easily estimated experimentally (see reference [1]).

Reaction Torque

The rotor drag gives rise to a torque on the vehicle airframe, with respect to CM. It is called reaction torque and is modeled by

$$\tau = k_\tau \omega^2 \tag{3}$$

where k_τ is the torque coefficient.



Remark: Similar to k_f, the coefficient k_τ also depends on the air density, the propeller geometry, and the flow regime. It can also be estimated experimentally (see reference [1]).

Other Aerodynamic Effects

Besides the above basic aerodynamic effects, there are others that are much more challenging to describe precisely. The main are:

- ground effect
- blade flapping
- variation due to stream velocity and angle of attack
- parasitic drag (form, skin, and interference)

The above effects have been well-investigated in the literature of full-scale helicopters (see reference [2]). Since it is difficult to precisely model the above effects, they will be often assumed as disturbances.

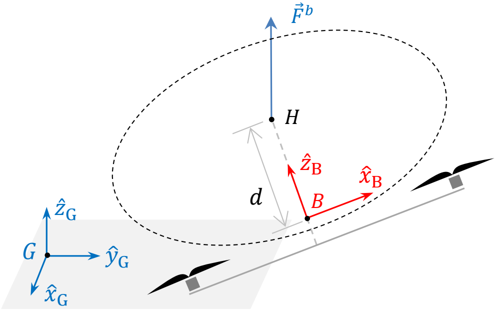
Aerostatic Balloon

Aerostatic Lift (see reference [4])

The balloon filled with a lifting gas (helium, hydrogen, hot air, etc.) generates a force $\vec{F}^b$ on the airframe. It is explained by the Archimedes' Principle:

$$\mathbf{F}_{\mathrm{G}}^{b} = F^{b}\mathbf{e}_3 \tag{4}$$

$$F^{b} = Vg\,\rho_{\mathrm{air}} \tag{5}$$

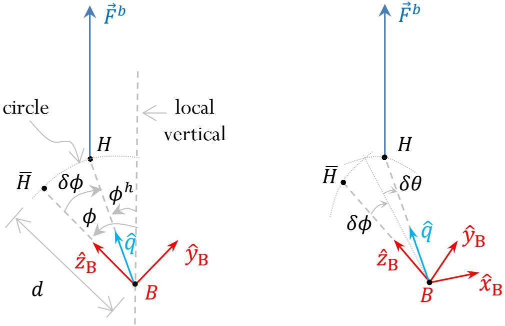


Aerostatic Balloon

Restoring Torque

Considering a flexible balloon-airframe connection, from the illustration:

$$\overrightarrow{T}^b = d\hat{q} \times \overrightarrow{F}^b \tag{6}$$



Aerostatic Balloon

Restoring Torque

The $\mathcal{S}_{\mathrm{B}}$ representation of $\overrightarrow{T}^b$ is

$$\mathbf{T}_{\mathrm{B}}^b = F^b d[\mathbf{q}_{\mathrm{B}}\times]\mathbf{D}^{\mathrm{B/G}}\mathbf{e}_3 \tag{7}$$

where $\mathbf{q}_{\mathrm{B}} = [\sin\delta\theta \;\; \sin\delta\phi\cos\delta\theta \;\; \cos\delta\phi\cos\delta\theta]^{\mathrm{T}}$, $\delta\phi \triangleq \phi^h - \phi$, $\delta\theta \triangleq \theta^h - \theta$, ϕ and θ are the roll and pitch angles corresponding to $\mathbf{D}^{\mathrm{B/G}}$, and ϕ^h and θ^h are the roll and pitch angles representing the attitude of $\hat{q}$ w.r.t. S_{G}.

The connection model can be

$$\ddot{\phi}^h + K^d\dot{\phi}^h + K^s\phi^h = K^s\phi, \tag{8}$$

$$\ddot{\theta}^h + K^d\dot{\theta}^h + K^s\theta^h = K^s\theta, \tag{9}$$

where K^d is a damping coefficient and K^s is a stiffness coefficient.

Aerostatic Balloon

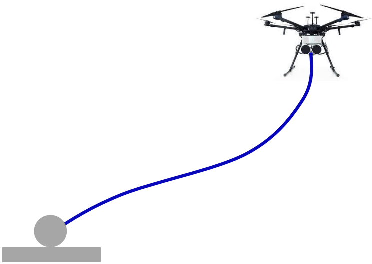
Remark:

In general, the mass and inertia matrix of the balloon (gas and envelop) need to be taken into account in the formulation of the MAV dynamic equations. This problem will be tackled later (in Chapter 5).

Tether

Introduction:

- Cable (tether) are useful for transfering data and electric energy.
- The cable dynamics ultimated produces an external force (tension) on the MAV airframe.
- Cable dynamics are in general modeled by either a partial differential equation or a series of point masses (lumped-mass model).



Tether

Lumped-Mass Model

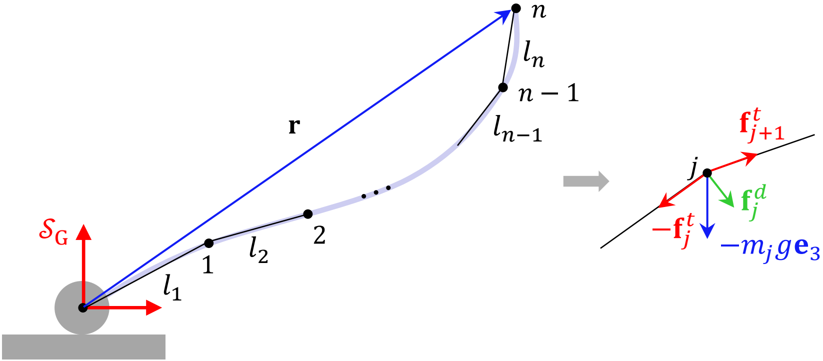
The cable is described as a series of point masses connected by frictionless hinges and straight elastic massless segments. The masses and forces are lumped at the end of the segments.

Assume that:

- The $\mathcal{S}_{\mathrm{G}}$ representations of the MAV position and velocity are known.
- The cable is always tensioned.
- The cable has a constant length.
- The cable has a cylindrical profile.
- The cable is not subject to tangential drag.

Tether

Lumped-Mass Model (see reference [3])



Lumped-Mass Model

Using the Newton's Second Law and the above free body diagram:

$$m_j \ddot{\mathbf{r}}_j = -m_j g \mathbf{e}_3 + \mathbf{f}^t_{j+1} - \mathbf{f}^t_j + \mathbf{f}^d_j \qquad j = 1, ..., n-1 \tag{10}$$

where m_j is the mass of the jth point, $\mathbf{r}_j$ is the $\mathcal{S}_{\mathrm{G}}$ position of m_j, $\mathbf{f}^t_j$ is the internal cable force at m_j, and $\mathbf{f}^d_j$ is the local normal drag.

Lumped-Mass Model

By considering viscoelastic cable segments, the internal force $\mathbf{f}_j^t$ can be modeled by

$$\mathbf{f}_j^t = \left(EA\varepsilon_j + C\dot{\varepsilon}_j\right) \frac{\mathbf{r}_j - \mathbf{r}_{j-1}}{\|\mathbf{r}_j - \mathbf{r}_{j-1}\|} \qquad j = 1, ..., n \tag{11}$$

where E is the Young's modulus, A is the cross-sectional area, C is the damping coefficient, L_j is the (constant) unstrained length of the jth segment, $l_j \triangleq \|\mathbf{r}_j - \mathbf{r}_{j-1}\|$, and

$$\varepsilon_j \triangleq \frac{l_j - L_j}{L_j} \tag{12}$$

$$\dot{\varepsilon}_j = \frac{\dot{l}_j}{L_j} \tag{13}$$

Tether

Lumped-Mass Model

By considering a cylindrical cable profile and neglecting the tangential drag, the aerodynamic (drag) force $\mathbf{f}_j^d$ can be modeled by

$$\mathbf{f}_j^d = \frac{1}{2}\rho d C_N L_j \|\mathbf{v}_j^{\perp}\| \mathbf{v}_j^{\perp} \qquad j = 1, ..., n-1 \tag{14}$$

where ρ is the air density, d is the cross-section diameter, C_N is the normal drag coefficient, and $\mathbf{v}_j^{\perp}$ is given by

$$\mathbf{v}_j^{\perp} = \quad \mathbf{v}_j - \left[\mathbf{v}_j^{\mathrm{T}} \frac{\mathbf{r}_j - \mathbf{r}_{j-1}}{\|\mathbf{r}_j - \mathbf{r}_{j-1}\|}\right] \frac{\mathbf{r}_j - \mathbf{r}_{j-1}}{\|\mathbf{r}_j - \mathbf{r}_{j-1}\|} \tag{15}$$

$$\mathbf{v}_j = \mathbf{v}_j^w - \dot{\mathbf{r}}_j \tag{16}$$

with $\mathbf{v}_j^w$ representing the wind velocity at node j in $\mathcal{S}_{\mathrm{G}}$.

Disturbances

Random Model

For control design purposes, all the external efforts (torque or force) that we cannot model precisely will be considered as disturbances.

Denote the disturbance force (in $\mathcal{S}_{\mathrm{G}}$) and the disturbance torque (in $\mathcal{S}_{\mathrm{B}}$) by $\mathbf{F}_{\mathrm{G}}^{d}$ and $\mathbf{T}_{\mathrm{B}}^{d}$, respectively. We are going to adopt the following models [1]:

$$\mathbf{F}_{\mathrm{G}}^{d} \sim \mathcal{U}_{\mathbb{D}^{f}} \tag{17}$$

$$\mathbf{T}_{\mathrm{B}}^{d} \sim \mathcal{U}_{\mathbb{D}^{t}} \tag{18}$$

where the supports $\mathbb{D}^{f} \in \mathbb{R}^{3}$ and $\mathbb{D}^{t} \in \mathbb{R}^{3}$ are generally assumed to be known, convex, and compact.

[1] $\mathcal{U}_{\mathbb{D}}$ denotes a multivariate uniform probability distribution with support $\mathbb{D}$.

Actuators . . .

Actuators

Rotor

It is composed by a brushless (three-phase) motor and its drive circuit.

The motor angular speed ω can be modeled by

$$\frac{\omega(s)}{\bar{\omega}(s)} = \frac{k^m}{\tau^m s + 1} \tag{19}$$

where $\bar{\omega}$ is the angular speed command, k^m is the motor gain, and τ^m is its time constant.

Actuators

Servomotor

It is commonly composed by a brushed dc motor in closed-loop by a potentiomenter.

Its angular displacement β can be modeled by

$$\frac{\beta(s)}{\bar{\beta}(s)} = \frac{k_1^s}{s^2 + k_2^s s + k_3^s} \tag{20}$$

where $\bar{\beta}$, and k_1^s, k_2^s, and k_3^s are coefficients.

References

[1] Mahony, R., Kumar, V., Corke, P. Multirotor Aerial Vehicle – Modeling, Estimation, and Control of Quadrotor. **IEEE Robotics & Automation Magazine**, 2012.

[2] Leishman, J. (2000). **Principles of Helicopter Aerodynamics (Cambridge Aerospace Series)**. Cambridge, MA: Cambridge Univ. Press.

[3] Williams, P. Cable Modeling Approximations for Rapid Simulation. **Journal of Guidance, Control, and Dynamics**, Vol. 40, No. 7, 2017.

[4] Santos, D.A., Cunha Jr, A. Flight control of a hexa-rotor airship: Uncertainty quantification for a range of temperature and pressure conditions. **ISA Transactions**, Vol. 93, 2019.