MP-282
Dynamic Modeling and Control of Multirotor Aerial Vehicles
Chapter 3: Resulting Control Efforts

Prof. Dr. Davi Antônio dos Santos
Instituto Tecnológico de Aeronáutica
www.professordavisantos.com

São José dos Campos - SP
2020

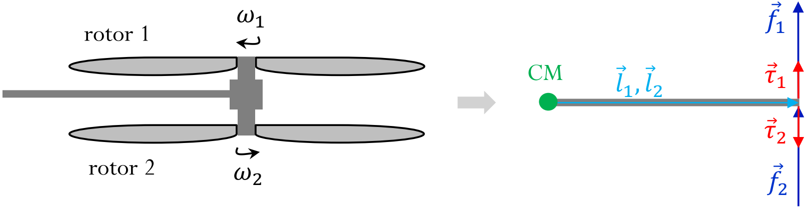
Contents

Notation and Concepts . . .

Notation and Concepts

Diagram of Control Efforts

- $\omega_i, \forall i$: angular speed of rotor i
- $\overrightarrow{f}_i, \forall i$: thurst provided by rotor i
- $\overrightarrow{\tau}_i, \forall i$: reaction torque provided by rotor i (contrary to ω_i)
- $\overrightarrow{l}_i$: rotor-CM arm



Notation and Concepts

Control Efforts for an Arbitrary MAV

Resulting Control Force:

$$\overrightarrow{F}^c = \sum_{i=1}^{n_r} \overrightarrow{f}_i \quad (1)$$

where n_r is the number of rotors.

Resulting Control Torque:

$$\overrightarrow{T}^c = \sum_{i=1}^{n_r} \overrightarrow{\tau}_i + \sum_{i=1}^{n_r} \overrightarrow{l}_i \times \overrightarrow{f}_i \quad (2)$$

The first sum in (2) corresponds to the moments of the reaction forces, while the second one represents the moments of the thrust forces.

Notation and Concepts

Relation Between Reaction Torque and Thrust

In Chapter 2, we described the magnitude of the reaction torque and thurst force of a rotor by

$$f_i = k_f \ \omega_i^2 \tag{3}$$

$$\tau_i = k_\tau \ \omega_i^2 \tag{4}$$

for $i = 1, ..., n_r$.

Dividing (4) by (3), we find $\tau_i = kf_i$, with

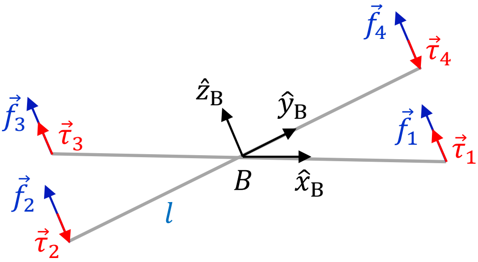
$$k \triangleq \frac{k_\tau}{k_f} \tag{5}$$

The parameter $k \in \mathbb{R}$ will be useful in this chapter.

Quadcopter Q+ . . .

Quadcopter Q+

Diagram of Control Efforts



Legend: l: arm length

Quadcopter Q+

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}^c_{\mathrm{B}}$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}^c_{\mathrm{B}} \end{bmatrix} = \mathbf{\Gamma}_{\mathrm{Q+}}\mathbf{f} \tag{6}$$

where $\mathbf{f} \triangleq [f_1 \;\; f_2 \; \dots \; f_4]^{\mathrm{T}}$ and

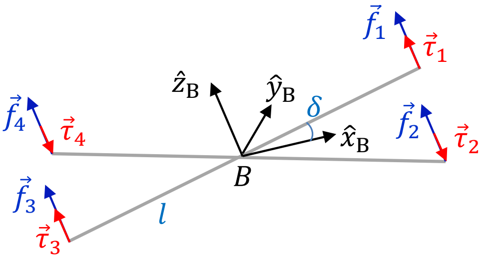
$$\mathbf{\Gamma}_{\mathrm{Q+}} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l & 0 & l \\ -l & 0 & l & 0 \\ k & -k & k & -k \end{bmatrix} \tag{7}$$

Proof: on the whiteboard.

Quadcopter QX …

Quadcopter QX

Diagram of Control Efforts



Legend: l: arm length
δ: (half) frontal separation angle

Quadcopter QX

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}^c_{\mathrm{B}}$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}^c_{\mathrm{B}} \end{bmatrix} = \mathbf{\Gamma}_{\mathrm{QX}} \mathbf{f} \tag{8}$$

where $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; \ldots \;\; f_4]^{\mathrm{T}}$ and

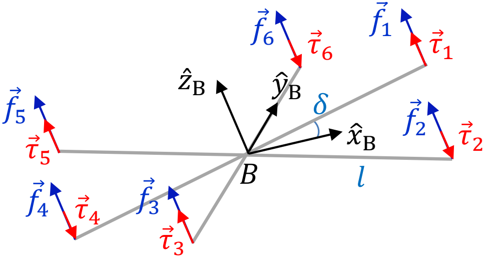
$$\mathbf{\Gamma}_{\mathrm{QX}} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ l \sin\delta & -l \sin\delta & -l \sin\delta & l \sin\delta \\ -l \cos\delta & -l \cos\delta & l \cos\delta & l \cos\delta \\ k & -k & k & -k \end{bmatrix} \tag{9}$$

Proof: on the whiteboard.

Hexacopter . . .

Hexacopter

Diagram of Control Efforts



Legend: l: arm length
δ: (half) frontal separation angle

Hexacopter

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}_\mathrm{B}^c$ in $\mathcal{S}_\mathrm{B}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}_\mathrm{B}^c \end{bmatrix} = \boldsymbol{\Gamma}_\mathrm{H}\mathbf{f} \tag{10}$$

where $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; \ldots \;\; f_6]^\mathrm{T}$ and

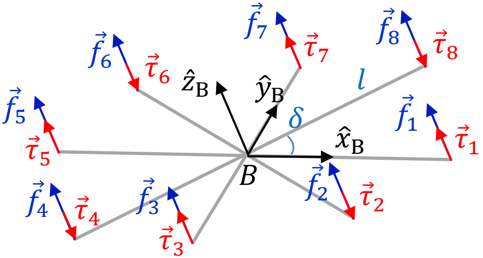
$$\boldsymbol{\Gamma}_\mathrm{H} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ l\sin\delta & -l\sin\delta & -l & -l\sin\delta & l\sin\delta & l \\ -l\cos\delta & -l\cos\delta & 0 & l\cos\delta & l\cos\delta & 0 \\ k & -k & k & -k & k & -k \end{bmatrix} \tag{11}$$

Proof: on the whiteboard.

Octacopter . . .

Octacopter

Diagram of Control Efforts



Legend: l: arm length
δ: frontal separation angle

Octacopter

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}^c_{\mathrm{B}}$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}^c_{\mathrm{B}} \end{bmatrix} = \mathbf{\Gamma}_{\mathrm{O}}\mathbf{f} \tag{12}$$

where $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; \dots \;\; f_8]^{\mathrm{T}}$ and

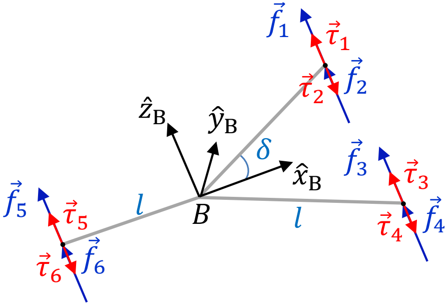
$$\mathbf{\Gamma}_{\mathrm{O}} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -l\sin\delta & -l & -l\sin\delta & 0 & l\sin\delta & l & l\sin\delta \\ -l & -l\cos\delta & 0 & l\cos\delta & l & l\cos\delta & 0 & -l\cos\delta \\ k & -k & k & -k & k & -k & k & -k \end{bmatrix} \tag{13}$$

Proof: left as an exercise.

Tricopter with double rotors . . .

Tricopter Y6

Diagram of Control Efforts



Legend: l: arm length

δ: (half) frontal separation angle

Tricopter Y6

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}_{\mathrm{B}}^c$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}_{\mathrm{B}}^c \end{bmatrix} = \boldsymbol{\Gamma}_{\mathrm{Y6}}\mathbf{f} \tag{14}$$

where $\mathbf{f} \triangleq [f_1 \; f_2 \; \ldots \; f_6]^{\mathrm{T}}$ and

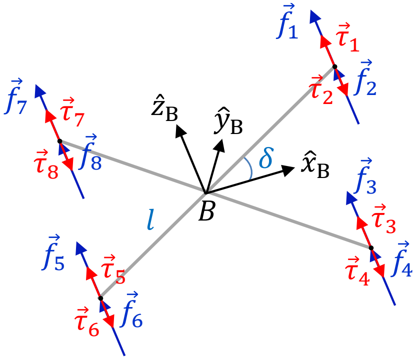
$$\boldsymbol{\Gamma}_{\mathrm{Y6}} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ l\sin\delta & l\sin\delta & -l\sin\delta & -l\sin\delta & 0 & 0 \\ -l\cos\delta & -l\cos\delta & -l\cos\delta & -l\cos\delta & l & l \\ k & -k & k & -k & k & -k \end{bmatrix} \tag{15}$$

Proof: on the whiteboard.

Quadcopter with double rotors . . .

Quadcopter X8

Diagram of Control Efforts



Legend: l: arm length
δ: (half) frontal separation angle

Quadcopter X8

Resulting Control Efforts

Resulting force magnitude F^c and resulting torque $\mathbf{T}^c_{\mathrm{B}}$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F^c \\ \mathbf{T}^c_{\mathrm{B}} \end{bmatrix} = \boldsymbol{\Gamma}_{\mathrm{X8}}\mathbf{f} \tag{16}$$

where $\mathbf{f} \triangleq [f_1 \ f_2 \ \dots \ f_8]^{\mathrm{T}}$, $\mathrm{s} \equiv \sin$, $\mathrm{c} \equiv \cos$, and

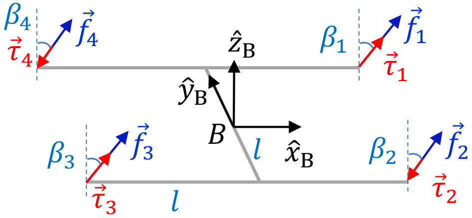
$$\boldsymbol{\Gamma}_{\mathrm{X8}} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ l\mathrm{s}\delta & l\mathrm{s}\delta & -l\mathrm{s}\delta & -l\mathrm{s}\delta & -l\mathrm{s}\delta & -l\mathrm{s}\delta & l\mathrm{s}\delta & l\mathrm{s}\delta \\ -l\mathrm{c}\delta & -l\mathrm{c}\delta & -l\mathrm{c}\delta & -l\mathrm{c}\delta & l\mathrm{c}\delta & l\mathrm{c}\delta & l\mathrm{c}\delta & l\mathrm{c}\delta \\ k & -k & k & -k & k & -k & k & -k \end{bmatrix} \tag{17}$$

Proof: left as an exercise.

Quadcopter with longitudinal-vectoring rotors . . .

Quadcopter with longitudinal-vectoring rotors

Diagram of Control Efforts



Legend: l: arm length
β_i: vectoring angles

Resulting Control Efforts

Resulting force components F_1^c, F_3^c and resulting torque $\mathbf{T}_{\mathrm{B}}^c$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} F_1^c \\ F_3^c \\ \mathbf{T}_{\mathrm{B}}^c \end{bmatrix} = \mathbf{\Gamma}_{\mathrm{LV4}}(\beta_1, \dots, \beta_4)\mathbf{f} \tag{18}$$

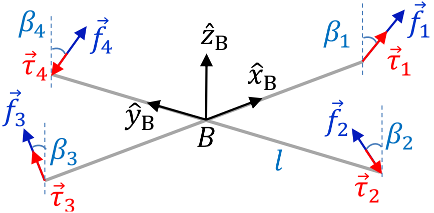
where $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; \dots \;\; f_4]^{\mathrm{T}}$, $\mathrm{s} \equiv \sin$, $\mathrm{c} \equiv \cos$, and

$$\mathbf{\Gamma}_{\mathrm{LV4}} \triangleq \begin{bmatrix} \mathrm{s}\beta_1 & \mathrm{s}\beta_2 & \mathrm{s}\beta_3 & \mathrm{s}\beta_4 \\ \mathrm{c}\beta_1 & \mathrm{c}\beta_2 & \mathrm{c}\beta_3 & \mathrm{c}\beta_4 \\ l\mathrm{c}\beta_1 + k\mathrm{s}\beta_1 & -l\mathrm{c}\beta_2 - k\mathrm{s}\beta_2 & -l\mathrm{c}\beta_3 + k\mathrm{s}\beta_3 & l\mathrm{c}\beta_4 - k\mathrm{s}\beta_4 \\ -l\mathrm{c}\beta_1 & -l\mathrm{c}\beta_2 & l\mathrm{c}\beta_3 & l\mathrm{c}\beta_4 \\ -l\mathrm{s}\beta_1 + k\mathrm{c}\beta_1 & l\mathrm{s}\beta_2 - k\mathrm{c}\beta_2 & l\mathrm{s}\beta_3 + k\mathrm{c}\beta_3 & -l\mathrm{s}\beta_4 - k\mathrm{c}\beta_4 \end{bmatrix}$$

Proof: on the whiteboard.

Quadcopter with transversal-vectoring rotors . . .

Diagram of Control Efforts



Legend: l: arm length
β_i: vectoring angles

Quadcopter with transversal-vectoring rotors

Resulting Control Efforts

Resulting force $\mathbf{F}^c_{\mathrm{B}}$ and resulting torque $\mathbf{T}^c_{\mathrm{B}}$ in $\mathcal{S}_{\mathrm{B}}$:

$$\begin{bmatrix} \mathbf{F}^c_{\mathrm{B}} \\ \mathbf{T}^c_{\mathrm{B}} \end{bmatrix} = \mathbf{\Gamma}_{\mathrm{TV4}}(\beta_1, ..., \beta_4)\mathbf{f} \tag{19}$$

where $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; ... \;\; f_4]^{\mathrm{T}}$, $\mathrm{s} \equiv \sin$, $\mathrm{c} \equiv \cos$, and

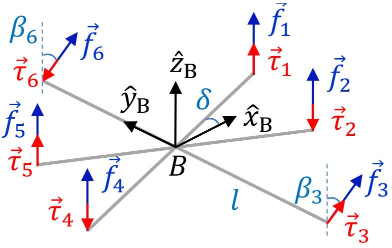
$$\mathbf{\Gamma}_{\mathrm{TV4}} \triangleq \begin{bmatrix} 0 & -\mathrm{s}\beta_2 & 0 & \mathrm{s}\beta_4 \\ -\mathrm{s}\beta_1 & 0 & \mathrm{s}\beta_3 & 0 \\ \mathrm{c}\beta_1 & \mathrm{c}\beta_2 & \mathrm{c}\beta_3 & \mathrm{c}\beta_4 \\ 0 & k\mathrm{s}\beta_2 - l\mathrm{c}\beta_2 & 0 & -k\mathrm{s}\beta_4 + l\mathrm{c}\beta_4 \\ -k\mathrm{s}\beta_1 - l\mathrm{c}\beta_1 & 0 & k\mathrm{s}\beta_3 + l\mathrm{c}\beta_3 & 0 \\ k\mathrm{c}\beta_1 - l\mathrm{s}\beta_1 & -k\mathrm{c}\beta_2 - l\mathrm{s}\beta_2 & k\mathrm{c}\beta_3 - l\mathrm{s}\beta_3 & -k\mathrm{c}\beta_4 - l\mathrm{s}\beta_4 \end{bmatrix}$$

Proof: left as an exercise.

Hexacopter with two vectoring rotors . . .

Hexacopter with two vectoring rotors

Diagram of Control Efforts

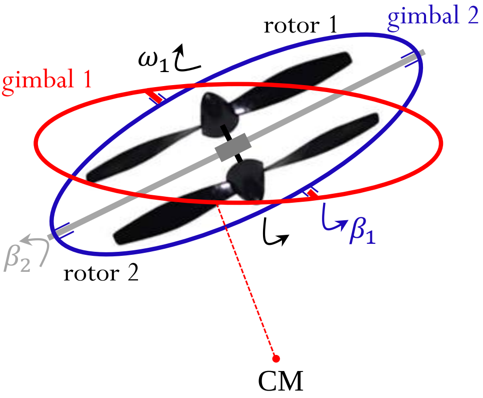


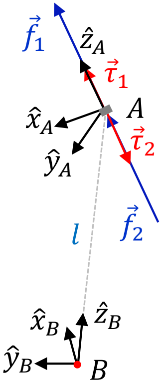
Homework: Obtain the $\mathcal{S}_{\mathrm{B}}$ representation of the control efforts.

Birotor on a 2-DOF gimbal . . .

Birotor on a 2-DOF guimbal

Diagram of Control Efforts





Homework: Obtain the $\mathcal{S}_\mathrm{B}$ representation of the control efforts.

Thanks!