# MP-208
# Optimal Filtering with Aerospace Applications
## Chapter 1: Introduction

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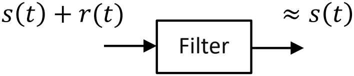
# Preliminary Definitions . . .

# Preliminary Defintions: What is filtering?

In classical signal processing:

Filtering is to separate signal and noise by their frequency.



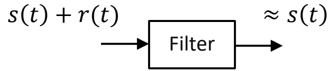

Examples:

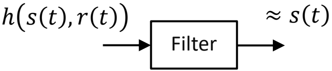
Low-pass filter, high-pass filter, band-pass filter, and band-rejection filter.

# Preliminary Definitions: What is filtering?

In statistical signal processing:

Filtering is to separate signal from noise by their statistical properties. We are interested in this type of filtering!



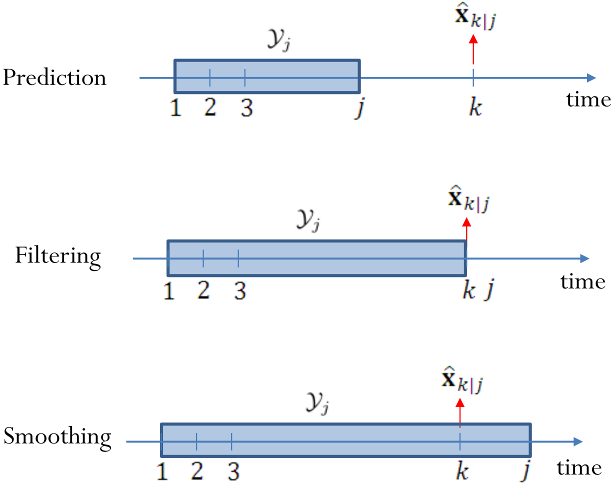




Examples:

Wiener filter, Kolmogorov filter, Kalman filter, Bucy filter, etc.

# Preliminary Definitions: What is filtering?

Three types of state estimation: prediction, filtering, and smoothing...



# Preliminary Definitions: Why optimal?

Optimal design of filters:

- In engineering, we are always interested in optimal solutions.
- We are looking for optimality in the following alternative senses:
  - Minimum mean square error (MMSE).
  - Maximum *a posteriori* probability (MAP).
  - Least squares (LS).
  - Maximum likelihood (ML).

# History . . .

# History

The Optimal Filtering Theory has been constructed in the following sequence:

- 1795: Johann Carl Friedrich Gauss devised the Least-Squares method for estimating the Ceres' orbit.
- 1940's: Wiener/Kolmogorov Filter to separate signal from noise using the MMSE criterion. They used a frequency-domain approach.
- 1950's: Attempts to extend Wiener/Kolmogorov filters for non-stationary and multivariate signals.
- 1960: Kalman-Bucy Filter was introduced as the new approach to tackle and extend the Weiner and Kolmogorov problems for non-stationary and multivariate signals.
- 1960-1970: Numerous applications in satellite orbit determination as well as in attitude determination and navigation of aircraft, ship, rocket, etc.

# History

- 1960-1970: Optimal Nonlinear Filtering developed mainly by Stratonovich (Russia) and consolidated by Kushner (EUA).
- 1990-2010: Particle Filters or sequential Monte Carlo methods.
- 1990-2010: More approximations of the Kalman filter for nonlinear systems: unscented Kalman filter, cubature Kalman filter, ensemble Kalman filter, etc.
- 2010-: Some hybrid schemes involving Kalman filter and machine learning.

Aerospace Applications. . .

# Aerospace Applications

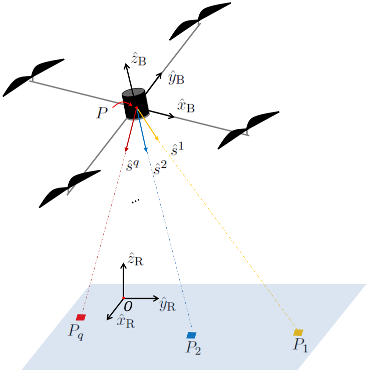
Regarding the dynamic nature of the quantity we want to estimate, we can distinguish between two types of estimation:

- Parameter estimation: the parameters are quantities that characterize the system of interest. Their values are assumed to be constant or smoothly time-varying.
- State estimation: The states are time-varying signals that describe the system dynamics.

# Aerospace Applications

## Image-Based Attitude Determination:

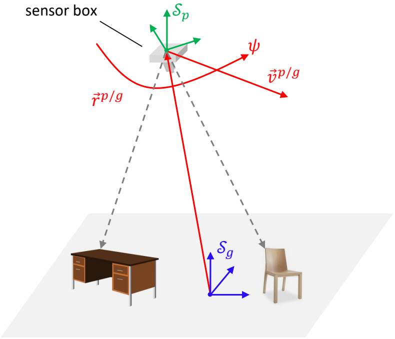
The three-dimensional attitude of an aerospace vehicle can be determined by optimal filtering using: (1) an attitude kinematic model together with rate-gyro measurements; (2) a set of vector measurements; (3) a map of landmarks; and (4) estimates/measurements of the vehicle's position.



# Aerospace Applications

## Image-Based Navigation:

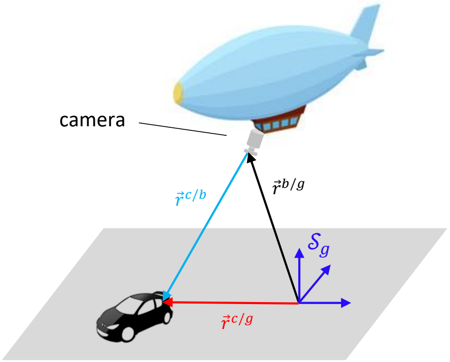
The navigation (position, velocity, and attitude estimation) of an aerospace vehicle can be realized by optimal filtering using: (1) position, velocity, and attitude kinematic models together with measurements taken from rate-gyros and accelerometers; (2) a set of measurements of landmark relative positions; and (3) a map of landmarks.



# Aerospace Applications

## Image-Based Object Tracking:

An object can be tracked by optimal filtering from an aerial vehicle equipped with a navigation (localization) system and a camera. For this end, it is necessary to know: (1) a kinematic model for describing the object motion; (2) measurements of the object relative position.



# Aerospace Applications

In aerospace systems, the most frequent applications of parameter estimation are:

- Calibration of navigation sensors.
- System identification.

Example. . .

# An Warm-up Example

Height and Vertical Velocity Estimation:

Consider a multirotor aerial vehicle (MAV) equipped with an ultrasonic sensor for measuring its height $h_k$ at each discrete-time instant $k$. Assume that the sensor is noise-free and denote the variable representing its measure at $k > 0$ by $y_k$. Consider a sampling period of $T = 0.1$ s.

1. Obtain a dynamic model of the plant in a discrete-time state-space representation.
2. Design a discrete-time Luenberger observer, with eigenvalues $\lambda_1 = 0.1$ and $\lambda_2 = 0.1$, for estimating the height $h_k$ and the vertical velocity $\dot{h}_k$ using $y_k, k > 0$.
3. Implement and test the designed observer using a MATLAB script.

# References. . .

# References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